Derivation and numerical solution of SDE in Neuroscience

Diana Morales[1], Bingyue Su[1], Renjun Zhu[1]

May 2, 2019

Table of Contents

Background

- We study Modelling of Neuroscience problems. Consider the model as: $\tau \cdot \frac{d\vec{y}}{dx} = -\vec{y} + J\vec{y} + \vec{x}$

Background

- We study Modelling of Neuroscience problems. Consider the model as: $\tau \cdot \frac{d\vec{y}}{dx} = -\vec{y} + J\vec{y} + \vec{x}$
  - Where $\vec{y}(t)$ is a vector of neurons' "activity"(i.e. firing rates).
  - and $\vec{x}(t)$ is a vector of neurons' external synaptic inputs from outside the local network.
  - J is an N by N matrix representing synaptic weights and the time-course of synaptic filters.
  - τ is a constant.

Background

- We study Modelling of Neuroscience problems. Consider the model as: $\tau \cdot \frac{d\vec{y}}{dx} = -\vec{y} + J\vec{y} + \vec{x}$
  - Where $\vec{y}(t)$ is a vector of neurons' "activity"(i.e. firing rates).
  - and $\vec{x}(t)$ is a vector of neurons' external synaptic inputs from outside the local network.
  - J is an N by N matrix representing synaptic weights and the time-course of synaptic filters.
  - τ is a constant.

- **Goal**: find the solution of $\vec{y}(t)$ and study the Cross-Special Density between two stationary processes, denote as $< \vec{x}(t), \vec{y}(t) >$.

Some Definition:

Stationarity: A process $\vec{x}(t)$ is stationary if its statistics are invariant to time translation. i.e. $\vec{x}(t)$ has the same statistics as $\vec{y}(t) = \vec{x}(t - t_0)$

Some Definition:

Stationarity: A process $\vec{x}(t)$ is stationary if its statistics are invariant to time translation. i.e. $\vec{x}(t)$ has the same statistics as $\vec{y}(t) = \vec{x}(t - t_0)$

Statistics: In this project, we are interested in statistics like mean and variance. Defined $\overline{\vec{x}} = \mathbb{E}[\vec{x}(t)]$ to be the mean of a stationary process $\vec{x}(t)$, and this does not depend on t.

Some Definition:

Stationarity: A process $\vec{x}(t)$ is stationary if its statistics are invariant to time translation. i.e. $\vec{x}(t)$ has the same statistics as $\vec{y}(t) = \vec{x}(t - t_0)$

Statistics: In this project, we are interested in statistics like mean and variance. Defined $\overline{\vec{x}} = \mathbb{E}[\vec{x}(t)]$ to be the mean of a stationary process $\vec{x}(t)$, and this does not depend on t.

Cross-Covairance: For two stationary processe, define $C_{xy}(\tau) = cov(\vec{x}(t), \vec{y}(t+\tau)) = \mathbb{E}[\vec{x}(t)\vec{y}(t+\tau)] - \overline{\vec{x}} \cdot \overline{\vec{y}}$ to be their cross-covariance function. Note: auto-covariance function...

Some Definition:

Stationarity: A process $\vec{x}(t)$ is stationary if its statistics are invariant to time translation. i.e. $\vec{x}(t)$ has the same statistics as $\vec{y}(t) = \vec{x}(t - t_0)$

Statistics: In this project, we are interested in statistics like mean and variance. Defined $\overline{\vec{x}} = \mathbb{E}[\vec{x}(t)]$ to be the mean of a stationary process $\vec{x}(t)$, and this does not depend on t.

Cross-Covairance: For two stationary processe, define $C_{xy}(\tau) = cov(\vec{x}(t), \vec{y}(t+\tau)) = \mathbb{E}[\vec{x}(t)\vec{y}(t+\tau)] - \overline{\vec{x}} \cdot \overline{\vec{y}}$ to be their cross-covariance function. Note: auto-covariance function...

Cross-spectral density(CSD): The CSD between two stationary processes is the Fourier Transform of their cross-covariance function. i.e. $< \vec{x}(t), \vec{y}(t) > (f) = \widetilde{C_{xy}}(f)$.

Solution: $\vec{y}(t)$

Solution of the Model: $\tau \cdot \frac{d\vec{y}}{dt} = -\vec{y} + J\vec{y} + \vec{x}$

Solution in terms of Convolution: $\vec{y}(t) = A * (J\vec{y}(t) + \vec{x}(t))$.

- A is a matrix kernel-matrix only contains diagonal entries such as $a(t) = \frac{1}{\tau}e^{-t/\tau}H(t)$.
- H(t) is the Heaviside step function
- J is a random square matrix with size N.

Each component satisfies $\vec{y}_j(t) = a * (J\vec{y}_j(t) + \vec{x}_j(t))$.
For **Stationrity**, we need to assume $J - Id$ with $\text{Re}\{\lambda\} < 0$.

CSD: $<\vec{y}(t),\vec{y}(t)>$

CSD of the Model: $\tau \cdot \frac{d\vec{y}}{dx} = -\vec{y} + J\vec{y} + \vec{x}$

Given $\vec{y}(t) = A * (J\vec{y}(t) + \vec{x}(t))$;

Then,

$$\begin{aligned} <\vec{y},\vec{y}> &= <A*(J\vec{y}+\vec{x}), A*(J\vec{y}+\vec{x})> \\ &= \ldots \\ &= (\tilde{A}^{-1} - J)^{-1} <\vec{x},\vec{x}> (\tilde{A}^{-1} - J)^{-*} \end{aligned}$$

Properties: Let K(t) be a time-dependent matrix, we called it a matrix kernel:
$<K*\vec{x},\vec{y}>=\tilde{K}<\vec{x},\vec{y}>$
$<\vec{x},K*\vec{y}>=<\vec{x},\vec{y}>\tilde{K}^*$, where $\tilde{K}^*$ is the conjugate-transpose.

Solution of a Very Simple Case in 1-D: $y(t)$

Solution of the Model: $\tau \cdot \frac{dy}{dt} = -y + x$

Solution in terms of Convolution: $y(t) = a * x(t)$.

$$\text{RHS} = -y(t) + x(t) = -\int_{-\infty}^{\infty} x(s) \cdot a(t-s) ds + x(t)$$

$$\begin{aligned} LHS = \tau \cdot \frac{dy}{dt} &= \int_{-\infty}^{\infty} x(s) \cdot \frac{-1}{\tau} e^{\frac{-(t-s)}{\tau}} H(t-s) ds \\ &+ \int_{-\infty}^{\infty} x(s) \cdot e^{\frac{-(t-s)}{\tau}} \delta(t-s) ds \\ &= \int_{-\infty}^{\infty} -x(s) \cdot a(t-s) ds + x(t) \\ &= -y(t) + x(t) \\ &= RHS \end{aligned}$$

CSD: $< y(t), < y(t) >$

CSD of the Model: $\tau \cdot \frac{d\vec{y}}{dt} = -y + x$

Given $y(t) = a * x(t))$;

Then,

$$\begin{aligned} < y, y > &= < a * x(t), a * x(t) > (f) \\ &= \tilde{a} < x(t), x(t) > \tilde{a}^*(f) \end{aligned}$$

since $a(t) = \frac{1}{\tau} e^{-t/\tau} H(t)$, we have $\tilde{a}(f) = \frac{1}{1+2\pi i f \tau}$. Often time, we are interested in lower-frequency CSD, i.e. f = 0, so $\tilde{a}(0) = 1$.

Statistics of the $E[<\vec{y},\vec{y}>]$

Write $<\vec{x},\vec{x}>$ in terms of $<\vec{y},\vec{y}>$ since we know how to compute the average of $<\vec{x},\vec{x}>$:

$$\begin{aligned}<\vec{x},\vec{x}> &= (\hat{A}^{-1}-J)<\vec{y},\vec{y}>(\hat{A}^{-1}-J)^* \\ &= (\hat{A}^{-1}-J)<\vec{y},\vec{y}>(\hat{A}^{-*}-J^*)\end{aligned}$$

Statistics of the E$[< \vec{y}, \vec{y} >]$

Write $< \vec{x}, \vec{x} >$ in terms of $< \vec{y}, \vec{y} >$ since we know how to compute the average of $< \vec{x}, \vec{x} >$:

$$\begin{aligned} < \vec{x}, \vec{x} > &= (\hat{A}^{-1} - J) < \vec{y}, \vec{y} > (\hat{A}^{-1} - J)^* \\ &= (\hat{A}^{-1} - J) < \vec{y}, \vec{y} > (\hat{A}^{-*} - J^*) \end{aligned}$$

$$\begin{aligned} \mathbb{E}[< \vec{x}, \vec{x} >] &= \hat{A}^{-1} \mathbb{E}[< \vec{y}, \vec{y} >] \hat{A}^{-*} \\ &- \hat{A}^{-1} \cdot \mathbb{E}[< \vec{y}, \vec{y} > J^*] \\ &- \mathbb{E}[J \cdot < \vec{y}, \vec{y} >] \hat{A}^{-*} \\ &+ \mathbb{E}[J < \vec{y}, \vec{y} > J^*] \end{aligned}$$

Statistics of the E[$<\vec{y},\vec{y}>$]

Write $<\vec{x},\vec{x}>$ in terms of $<\vec{y},\vec{y}>$ since we know how to compute the average of $<\vec{x},\vec{x}>$:

$$
\begin{aligned}
<\vec{x},\vec{x}> &= (\hat{A}^{-1} - J) <\vec{y},\vec{y}> (\hat{A}^{-1} - J)^* \\
&= (\hat{A}^{-1} - J) <\vec{y},\vec{y}> (\hat{A}^{-*} - J^*)
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}[<\vec{x},\vec{x}>] &= \hat{A}^{-1}\,\mathbb{E}[<\vec{y},\vec{y}>]\hat{A}^{-*} \\
&- \hat{A}^{-1}\cdot\mathbb{E}[<\vec{y},\vec{y}> J^*] \\
&- \mathbb{E}[J\cdot <\vec{y},\vec{y}>]\hat{A}^{-*} \\
&+ \mathbb{E}[J <\vec{y},\vec{y}> J^*]
\end{aligned}
$$

GOAL: We need to figure out $\mathbb{E}[<\vec{y},\vec{y}> J^*]$, $\mathbb{E}[J <\vec{y},\vec{y}>]$, and $\mathbb{E}[J <\vec{y},\vec{y}> J^*]$.

Solving for the Expectations

For the expectation of a matrix, we only need to figure out each entry of a matrix $[.]_{jk}$.

Solving for the Expectations

For the expectation of a matrix, we only need to figure out each entry of a matrix $[.]_{jk}$.

$$
\begin{aligned}
\mathbb{E}[<\vec{x},\vec{x}>]_{jk} &= \hat{A}_{jj}^{-1}\,\mathbb{E}[<\vec{y},\vec{y}>]_{jk}\hat{A}_{kk}^{-*} \\
&- \hat{A}_{jj}^{-1}\cdot\mathbb{E}[<\vec{y},\vec{y}>J^*]_{jk} \\
&- \mathbb{E}[J\cdot<\vec{y},\vec{y}>]_{jk}\hat{A}_{jk}^{-*} \\
&+ \mathbb{E}[J<\vec{y},\vec{y}>J^*]_{jk}
\end{aligned}
$$

Solving for the Expectations

$$\mathbb{E}[< \vec{y}, \vec{y} > J^*]_{jk} = \mathbb{E}[< \vec{y}, J\vec{y} >]_{jk} \approx (N-1) \cdot \overline{< \vec{y}, \vec{y} >} \cdot \overline{J^*} + \overline{\{\vec{y}, \vec{y}\}} \cdot \overline{J^*}$$

$$\mathbb{E}[J \cdot < \vec{y}, \vec{y} >]_{jk} = \mathbb{E}[< J\vec{y}, \vec{y} >]_{jk} \approx (N-1) \cdot \overline{J} \cdot \overline{< \vec{y}, \vec{y} >} + \overline{J} \cdot \overline{\{\vec{y}, \vec{y}\}}$$

$$\begin{aligned} \mathbb{E}[J < \vec{y}, \vec{y} > J^*]_{jk} &= \mathbb{E}[< J\vec{y}, J\vec{y} >]_{jk} \\ &\approx (N^2 - N) \cdot \overline{J} \cdot \overline{< \vec{y}, \vec{y} >} \cdot \overline{J^*} + N \cdot \overline{J} \cdot \overline{\{\vec{y}, \vec{y}\}} \cdot \overline{J^*} \end{aligned}$$

Let us consider an Erdos-Renyi network, where J is defined as:

$$J_{jk} = \begin{cases} \frac{j_0}{\sqrt{N}} & \text{with probability } p \\ 0 & \text{otherwise} \end{cases}$$

This represents randomly connected "inhibitory" or "negative" interactions in the network.

Solving for the Expectations

As $N \to \infty$, this becomes:

$$\begin{aligned}\overline{<\vec{x},\vec{x}>} &= \overline{<\vec{y},\vec{y}>}\Big[1-2(N-1)\cdot\frac{pj_0}{\sqrt{N}}+(N^2-N)\cdot\frac{p^2j_0^2}{N}\Big] \\ &- \overline{\{\vec{y},\vec{y}\}}\Big[2\cdot\frac{pj_0}{\sqrt{N}}-p^2j_0^2\Big]\end{aligned}$$

Solving for the Expectations

As $N \to \infty$, this becomes:

$$\begin{aligned}\overline{<\vec{x},\vec{x}>} &= \overline{<\vec{y},\vec{y}>}\Big[1-2(N-1)\cdot\frac{pj_0}{\sqrt{N}}+(N^2-N)\cdot\frac{p^2j_0^2}{N}\Big] \\ &- \overline{\{\vec{y},\vec{y}\}}\Big[2\cdot\frac{pj_0}{\sqrt{N}}-p^2j_0^2\Big]\end{aligned}$$

Now, we follow the same process to the diagonal part of $\mathbb{E}[<\vec{x},\vec{x}>]$, $\mathbb{E}[<\vec{x},\vec{x}>]_{jj}$. After simplifying:

Solving for the Expectations

As $N \to \infty$, this becomes:

$$\begin{aligned}\overline{<\vec{x},\vec{x}>} &= \overline{<\vec{y},\vec{y}>}\Big[1-2(N-1)\cdot\frac{pj_0}{\sqrt{N}}+(N^2-N)\cdot\frac{p^2j_0^2}{N}\Big] \\ &- \overline{\{\vec{y},\vec{y}\}}\Big[2\cdot\frac{pj_0}{\sqrt{N}}-p^2j_0^2\Big]\end{aligned}$$

Now, we follow the same process to the diagonal part of $\mathbb{E}[<\vec{x},\vec{x}>]$, $\mathbb{E}[<\vec{x},\vec{x}>]_{jj}$. After simplifying:

$$\begin{aligned}\overline{\{\vec{x},\vec{x}\}} &= \overline{\{\vec{y},\vec{y}\}}\Big[1-2\cdot\frac{pj_0}{\sqrt{N}}+p^2j_0^2\Big] \\ &- \overline{<\vec{y},\vec{y}>}\Big[2(N-1)\cdot\frac{pj_0}{\sqrt{N}}-(N^2-N)\cdot\frac{p^2j_0^2}{N}\Big]\end{aligned}$$

Solving for the Expectations

To see the long-term behaviour, we use $\mathcal{O}$ and o notation,

$$\begin{cases} \overline{<\vec{x},\vec{x}>} = \overline{<\vec{y},\vec{y}>} \cdot N(p^2 j_o^2) + o(N) - \overline{\{\vec{y},\vec{y}\}} \cdot (-p^2 j_o^2) + o(1) \\ \overline{\{\vec{x},\vec{x}\}} = \overline{\{\vec{y},\vec{y}\}} \cdot (1 + p^2 j_o^2) + o(1) - \overline{<\vec{y},\vec{y}>} \cdot N(-p^2 j_o^2) + o(N) \end{cases}$$

Hence, $\overline{<\vec{y},\vec{y}>} = -\frac{\overline{\{\vec{y},\vec{y}\}}}{N} + \frac{\overline{<\vec{x},\vec{x}>}}{p^2 j_0^2 N} + o(\frac{1}{N})$,

So we need to find a C according to $\overline{\{\vec{y},\vec{y}\}}$ and $\overline{<\vec{x},\vec{x}>}$ such that $\overline{<\vec{y},\vec{y}>} = \frac{C}{N} + o(\frac{1}{N}) \approx \mathcal{O}(\frac{1}{N})$

Identity Case: $<\vec{x},\vec{x}>= I_n$

With the identity case, only the diagonal contributes.
So, $\overline{<\vec{x},\vec{x}>} = 0$ and $\overline{\{\vec{x},\vec{x}\}} = 1$.

$$\begin{cases} 0 = \overline{<\vec{y},\vec{y}>} \cdot N(p^2 j_o^2) + o(N) + \overline{\{\vec{y},\vec{y}\}} \cdot (p^2 j_o^2) + o(1) \\ 1 = \overline{\{\vec{y},\vec{y}\}} \cdot (1 + p^2 j_o^2) + o(1) + \overline{<\vec{y},\vec{y}>} \cdot N(p^2 j_o^2) + o(N) \end{cases}$$

Then $\overline{\{\vec{y},\vec{y}\}} = 1$; together with $\overline{<\vec{x},\vec{x}>} = 0$, we have $C = -1$.
We expect $\overline{<\vec{y},\vec{y}>} = -\frac{1}{N} + o(\frac{1}{N})$

$<\vec{x},\vec{x}>\sim \mathcal{N}(\mu,\sigma)$ with Fixed Parameters

Since for each entry of $<\vec{x},\vec{x}>$ has expectation μ, we have $\overline{<\vec{x},\vec{x}>} = \overline{\{\vec{x},\vec{x}\}} = \mu$.

$$\begin{cases} \mu = \overline{<\vec{y},\vec{y}>}\cdot N(p^2 j_o^2) + o(N) + \overline{\{\vec{y},\vec{y}\}}\cdot(p^2 j_o^2) + o(1) \\ \mu = \overline{\{\vec{y},\vec{y}\}}\cdot(1+p^2 j_o^2) + o(1) + \overline{<\vec{y},\vec{y}>}\cdot N(p^2 j_o^2) + o(N) \end{cases}$$

then $\overline{\{\vec{y},\vec{y}\}} = 0$, so $C = \frac{\mu}{p^2 j_0^2}$.
We expect $\overline{<\vec{y},\vec{y}>} = \frac{C}{N} + o(\frac{1}{N}) = \frac{\mu}{p^2 j_0^2\cdot N} + o(\frac{1}{N})$

$<\vec{x},\vec{x}>\sim\mathcal{N}(N\mu,\sqrt{N}\sigma)$ with Non-Fixed Parameters

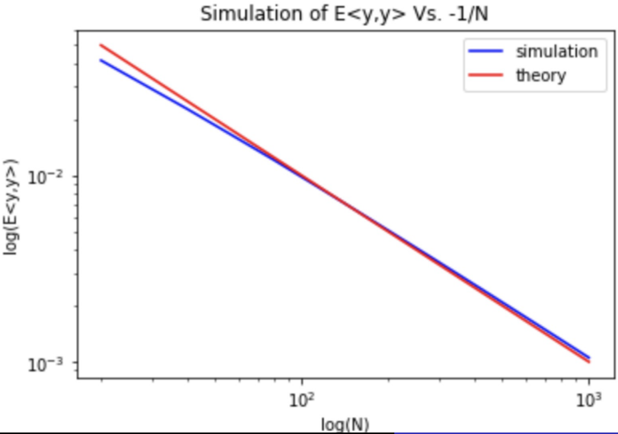
Since for each entry of $<\vec{x},\vec{x}>$ has expectation $N\mu$, comparing with the one with fixed-parameters, C would be $\frac{\mu}{p^2 j_0^2}\cdot N$.

$\overline{<\vec{y},\vec{y}>} = \frac{\mu}{p^2 j_0^2} + o(1)$

Identity Case

$<\vec{x},\vec{x}>= I_n$:

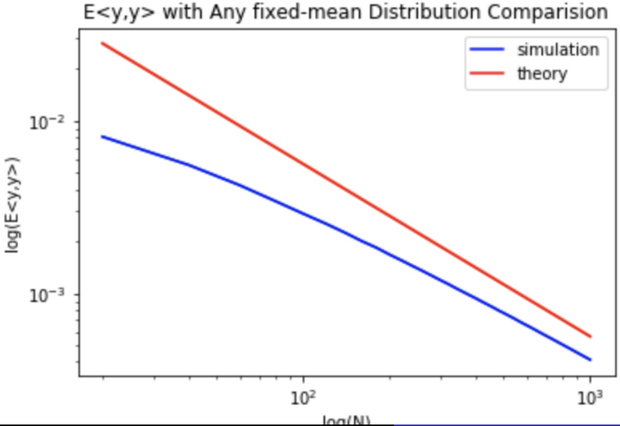
Theoretical value: $\overline{<\vec{y},\vec{y}>} = -\frac{1}{N} + o(\frac{1}{N})$



Normal Case With Fixed Parameters

$< \vec{x}, \vec{x} > \sim \mathcal{N}(\mu, \sigma^2)$:

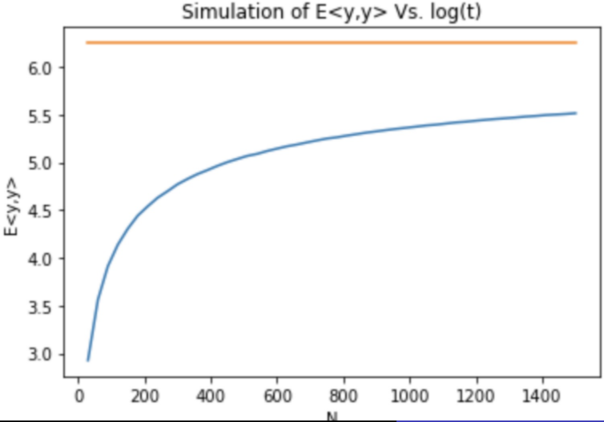
Theoretical value: $\overline{< \vec{y}, \vec{y} >} = \frac{C}{N} + o(\frac{1}{N}) = \frac{\mu}{p^2 j_0^2 \cdot N} + o(\frac{1}{N})$



Normal Case With Non-fixed Parameters

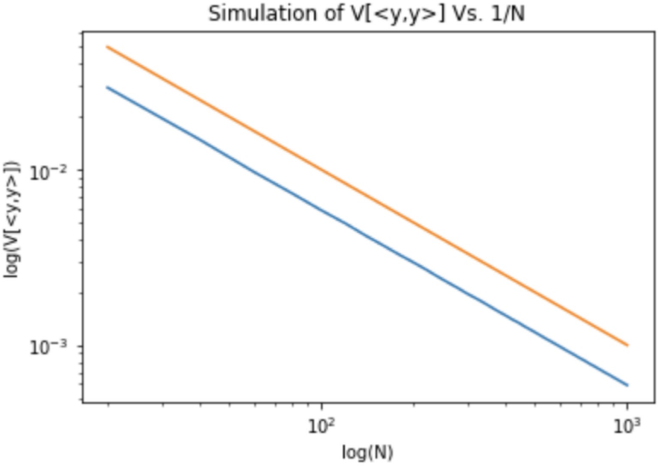
$<\vec{x},\vec{x}>\sim \mathcal{N}(N\mu, \sqrt{N}\sigma^2)$:

Theoretical value: $\overline{<\vec{y},\vec{y}>} = C + o(\frac{1}{N}) = \frac{\mu}{p^2 j_0^2} + o(1)$



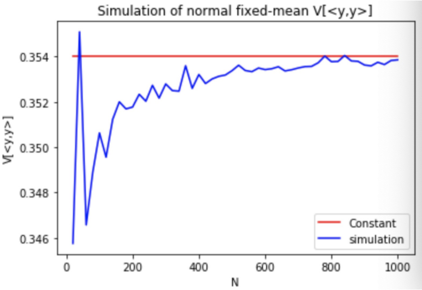
Identity Case

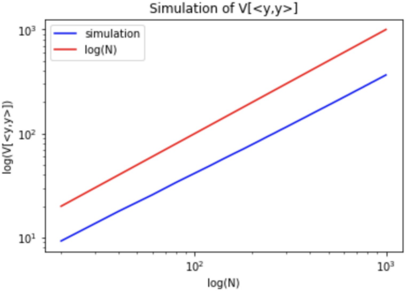
$< \vec{x}, \vec{x} >= I_n$:



Normal Case

$<\vec{x},\vec{x}>\sim \mathcal{N}(\mu,\sigma^2) or \mathcal{N}(N\mu,\sqrt{N}\sigma^2)$:





Numerically Simulate SDE:

$\frac{d\vec{y}}{dt} = F(\vec{y},t) + G(\vec{y},t)\frac{d\vec{W}}{dt}$, where $\vec{W}(t) \in \mathbb{R}^m$ is an m-dimensional Winer process, and $G : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^{n \times m}$

Numerically Simulate SDE:

$\frac{d\vec{y}}{dt} = F(\vec{y}, t) + G(\vec{y}, t)\frac{d\vec{W}}{dt}$, where $\vec{W}(t) \in \mathbb{R}^m$ is an m-dimensional Winer process, and $G : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^{n \times m}$

In our model, it would be:
$\vec{y}_{i+1} = \vec{y}_i + (J - I)\vec{y}_i \cdot dt + dw$, where $dW \sim \mathcal{N}(0, \sqrt{dt})$, i is the steps that we partition on .

Numerically Simulate SDE:

$\frac{d\vec{y}}{dt} = F(\vec{y}, t) + G(\vec{y}, t)\frac{d\vec{W}}{dt}$, where $\vec{W}(t) \in \mathbb{R}^m$ is an m-dimensional Winer process, and $G : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^{n\times m}$

In our model, it would be:
$\vec{y}_{i+1} = \vec{y}_i + (J - I)\vec{y}_i \cdot dt + dw$, where $dW \sim \mathcal{N}(0, \sqrt{dt})$, i is the steps that we partition on .

By Fourier transfer, I can get $lim_{\tau_0 \to \infty} \frac{cov(N_{yj}(\tau_0), N_{y_k}(\tau_0))}{\tau_0} =< y_j, y_k >$

Numerically Simulate SDE:

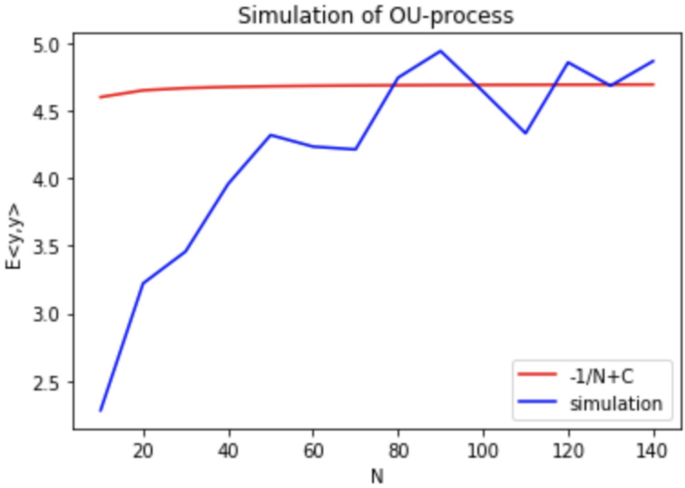
$\frac{d\vec{y}}{dt} = F(\vec{y}, t) + G(\vec{y}, t)\frac{d\vec{W}}{dt}$, where $\vec{W}(t) \in \mathbb{R}^m$ is an m-dimensional Winer process, and $G : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^{n\times m}$

In our model, it would be:
$\vec{y}_{i+1} = \vec{y}_i + (J - I)\vec{y}_i \cdot dt + dw$, where $dW \sim \mathcal{N}(0, \sqrt{dt})$, i is the steps that we partition on .

By Fourier transfer, I can get $lim_{\tau_0 \to \infty} \frac{cov(N_{yj}(\tau_0), N_{y_k}(\tau_0))}{\tau_0} =< y_j, y_k >$

In simulation, I can choose a large τ_0 to estimate $\mathbb{E}[< y_j, y_k >]$:
$\frac{cov(N_{y_j}(\tau_0), N_{y_k}(\tau_0))}{\tau_0} \approx < y_j, y_k >$

Numerically Simulate SDE:



In Summary:

Our Neuroscience Model:

$$\tau \cdot \frac{d\vec{y}}{dt} = -\vec{y} + J\vec{y} + \vec{x}$$

1. **Solution:** $\vec{y}(t) = A * (J\vec{y}(t) + \vec{x}(t))$.
2. **Cross-Spectral Density:**
   $< \vec{y}, \vec{y} >= (\tilde{A}^{-1} - J)^{-1} < \vec{x}, \vec{x} > (\tilde{A}^{-1} - J)^{-*}$
3. $\mathbb{E}[< \vec{y}, \vec{y} >] := \overline{< \vec{y}, \vec{y} >} = -\frac{\overline{\{\vec{y},\vec{y}\}}}{N} + \frac{\overline{<\vec{x},\vec{x}>}}{p^2 j_0^2 N} + o(\frac{1}{N})$
   - if $< \vec{x}, \vec{x} >= I$, then $\overline{< \vec{y}, \vec{y} >} \sim \mathcal{O}(\frac{1}{N})$.
   - if $< \vec{x}, \vec{x} >\sim \mathcal{N}(\mu, \sigma^2)$, then $\overline{< \vec{y}, \vec{y} >} \sim \mathcal{O}(\frac{1}{N})$.
   - if $< \vec{x}, \vec{x} >\sim \mathcal{N}(N\mu, \sqrt{N}\sigma^2)$, then $\overline{< \vec{y}, \vec{y} >} \sim \mathcal{O}(1)$.
4. **Simulations** confirms the Theoretical Derivations. ☺
5. **Investigate** on Variance and OU-Process.

Future Work: Derive Theoretical Variance and other Statistics.

Any Questions ??

Thank You ☺

Reference

- D. Dahmen, S. Grun, M. Diesmann, and M. Helias, *Two Types of Criticality in the brain*, (2017).
- C. Baker, C. Ebsch, I. Lampl, and R. Rosenbaum *The correlated state om balanced neronal networks*, (2019).